A Geometric Langlands Correspondence for Photonic Gauge Sectors

Peter De Ceuster

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Abstract

We propose a photonic adaptation of the Geometric Langlands program: a conjectural equivalence between categories associated to moduli of G-bundles with photonic boundary/coupling data (the "automorphic" side) and a spectral category built from Langlands-dual local systems augmented by photonic insertions (the "spectral" side). We define a moduli stack $\mathrm{Bun}_G^{\mathrm{ph}}(M)$ parameterizing principal G-bundles on a compact Riemann surface (or compactification of a photonic medium) together with prescribed photonic defect data and coupling forms; we define a photonic Hitchin/Local system space $\mathrm{Loc}^{\mathrm{ph}}_{{}^LG}(M)$ encoding dual-group local systems with gerbe-like photonic fluxes. Our main conjecture states a derived equivalence

$$\mathbf{D}\big(\mathcal{D}\text{-mod}(\mathrm{Bun}_G^{\mathrm{ph}}(M))\big) \simeq \mathbf{D}\big(\mathrm{QCoh}(\mathrm{Loc}^{\mathrm{ph}}_{{}^LG}(M))\big),$$

compatible with Hecke operators and photonic Hecke modifications. We work out the rank-one case $G = \mathrm{GL}_1$ (abelian Geometric Class Field Theory with photonic defects) explicitly and sketch how Fourier–Mukai transforms, twisted D-module techniques and spectral curve methods adapt to the photonic setting. We conclude with proposed tests (spectral measurements in photonic crystals, discrete lattice checks) and relations to stacky-sheaf viewpoints on photons and cohomological dualities.

Contents

1 Introduction

The Geometric Langlands program posits deep categorical correspondences between coherent sheaf-type categories on moduli of G-bundles (automorphic side) and categories associated to ${}^LG$-local systems (spectral side). Motivated by recent progress linking topological and categorical structures to engineered photonic systems, we formulate a photonic Geometric Langlands correspondence where both sides are enriched by photonic boundary conditions, defect insertions and higher-form (gerbe-like) fluxes modeling metamaterials and structured photonic media.

Photonic data appears in two natural guises:

- as *boundary/defect conditions* for gauge fields (e.g. surface impedances, chiral layers, waveguide terminations), and
- as *higher-form fluxes* or gerbe-curvatures encoding multi-form coupling in metamaterials.

Our new concept integrates these into a moduli-theoretic framework and conjectures a categorical equivalence parallel to classical Geometric Langlands, but with explicit photonic insertions. It is curious to use Langlands theorum in such a productive way and we believe such has potential.

2 Related work

Connecting higher stacks, twisted sheaves and physical systems with higher-form fluxes is realistic.We provide three concepts below that summarize the relevant technical foundations and motivate the photonic enrichments used throughout the abstract (see Sections 9, 10, 11). This should help advance understanding.

3 Setup and precise definitions

3.1 Base geometry

Let X be a smooth projective curve over $\mathbb{C}$ (a compact Riemann surface), or a compactified photonic medium modeled by such a curve. Fix a complex reductive group G and denote by ${}^LG$ its Langlands dual group.

3.2 Photonic boundary / defect data

We model photonic inhomogeneities (defects, layers, boundary impedances) by a finite set of marked points $D = \{x_1, \dots, x_m\} \subset X$ and by specifying, at each x_i, photonic coupling data:

$$\mathrm{ph}_i = (\ \mathcal{O}^{\mathrm{ph}}_{x_i},\ \alpha_i\ ),$$

where $\mathcal{O}^{\mathrm{ph}}_{x_i}$ is a local photonic boundary condition (e.g. a choice of Lagrangian subspace of boundary modes, impedance operator, or local gerbe datum) and α_i denotes a local coupling form (a germ of differential form / flux). These data may also include formal monodromy/ Stokes parameters when considering noncompact ends.

3.3 Moduli stack of photonic G-bundles

Definition 3.1. *The* moduli stack of photonic G-bundles

$$\mathrm{Bun}^{\mathrm{ph}}_G(X; D, \{\mathrm{ph}_i\})$$

parameterizes pairs $(\mathcal{E}, \Psi)$ *where* $\mathcal{E}$ *is a principal* G*-bundle on* X *and* Ψ *is photonic boundary data: a collection of local trivializations / modifications of* $\mathcal{E}$ *at points of* D *compatible with the prescribed* ph_i*, together with global coupling forms (gerbe-like fluxes)* $\kappa \in \Omega^\bullet(X)$ *satisfying specified quantization/integrality conditions.*

This stack is an algebraic (Artin) stack locally of finite type, with inertia reflecting automorphisms of bundles compatible with photonic insertions. For $G = \mathrm{GL}_1$ it reduces to a twisted Picard stack of line bundles with photonic data.

3.4 Hecke modifications with photonic insertions

Hecke operators are defined as correspondences modifying a G-bundle at a point. In the photonic setting, we require Hecke modifications to preserve or transform the photonic local data according to prescribed rules (e.g. induce shifts in local gerbe fluxes or change boundary impedance). Formally we have a Hecke correspondence

$$\mathrm{Hecke}^{\mathrm{ph}}_\lambda \subset \mathrm{Bun}^{\mathrm{ph}}_G \times X \times \mathrm{Bun}^{\mathrm{ph}}_G$$

parametrizing photonic Hecke modifications of type λ.

3.5 Spectral side: photonic local systems and Hitchin-like data

Define the *photonic spectral moduli*

$$\mathrm{Loc}^{\mathrm{ph}}_{{}^LG}(X; D, \{\mathrm{ph}_i\})$$

to be the moduli stack of ${}^LG$-local systems (flat ${}^LG$-connections) on $X \setminus D$ equipped with:

- prescribed local monodromy / Stokes data compatible with photonic insertions ph_i,
- gerbe-like photonic fluxes (classes in degree-2 or higher differential cohomology) that couple to the local system (twists of the moduli by differential characters).

Equivalently, one may work with a photonic Hitchin moduli space consisting of Higgs fields $(\mathcal{E}, \varphi)$ plus photonic flux parameters; the spectral curve acquires photonic deformations.

4 The new Conjecture

We now state the central conjecture.

Conjecture 4.1 (Photonic Geometric Langlands Correspondence)**.** *Let X be a smooth projective curve and let $D, \{\mathrm{ph}_i\}$ be photonic markings as above. There exists a canonical equivalence of triangulated (derived) categories*

$$\mathbf{D}\big(\mathcal{D}\text{-mod}\big(\mathrm{Bun}_G^{\mathrm{ph}}(X; D, \{\mathrm{ph}_i\})\big)\big) \;\simeq\; \mathbf{D}\big(\mathrm{QCoh}\big(\,\mathrm{Loc}^{\mathrm{ph}}_{{}^L G}(X; D, \{\mathrm{ph}_i\})\big)\big), \tag{4.1}$$

intertwining (a) photonic Hecke functors on the left with natural tensor/monodromy operations on the right, and (b) compatible with insertion/removal of photonic defects via natural operations on both sides. For G reductive this equivalence is expected to be t-exact with respect to appropriate perverse t-structures once photonic perversities are chosen.

Remark 4.2. *The equivalence* (4.1) *generalizes the usual categorical Geometric Langlands to include photonic twisting data: gerbe-type fluxes and boundary modifications. When all photonic data are trivialized, Conjecture 4.1 recovers the classical statement (subject to existing hypotheses).*

5 Rank-one toy case: $G = \mathrm{GL}_1$

The abelian case illustrates the conjecture concretely and is amenable to direct verification.

5.1 Automorphic side for $G = \mathrm{GL}_1$

For $G = \mathrm{GL}_1$ the moduli $\mathrm{Bun}^{\mathrm{ph}}_{\mathrm{GL}_1}$ is the Picard stack $\mathrm{Pic}^{\mathrm{ph}}(X; D, \{\mathrm{ph}_i\})$ of line bundles with photonic modifications: line bundles L together with local photonic framings / phases at D and global photonic flux twist (a degree-2 differential character). The category of D-modules on the Picard stack is equivalent (under Fourier–Mukai) to quasi-coherent sheaves on the dual torus of local systems.

5.2 Spectral side for ${}^L G = \mathrm{GL}_1$

Here ${}^L G = \mathrm{GL}_1$; $\mathrm{Loc}^{\mathrm{ph}}_{\mathrm{GL}_1}$ is the moduli of rank-one local systems with specified photonic monodromy and flux coupling. Concretely this is the character torus

$$\mathrm{Loc}^{\mathrm{ph}}_{\mathrm{GL}_1} \simeq \mathrm{Hom}\big(\pi_1(X \setminus D), \mathbb{C}^\times\big)_{\mathrm{ph}} \times \{\text{flux parameters}\},$$

a (twisted) algebraic torus.

5.3 Equivalence and matching of eigenvalues

In the abelian Fourier–Mukai framework one has a canonical integral transform (the geometric class field theory transform, adapted to photonic twist):

$$\Phi^{\mathrm{ph}}_{\mathrm{FM}} : \; \mathbf{D}\big(\mathcal{D}\text{-mod}(\mathrm{Pic}^{\mathrm{ph}})\big) \xrightarrow{\simeq} \mathbf{D}\big(\mathrm{QCoh}(\mathrm{Loc}^{\mathrm{ph}}_{\mathrm{GL}_1})\big).$$

Hecke eigen-D-modules correspond to skyscraper (character) sheaves on $\mathrm{Loc}^{\mathrm{ph}}_{\mathrm{GL}_1}$; the photonic Hecke operator acts by shifting the character by the local photonic monodromy, matching spectral eigenvalues. Thus Conjecture 4.1 holds for $G = \mathrm{GL}_1$ upon verifying the modified Fourier–Mukai kernel incorporates the photonic gerbe twist (a straightforward modification of the classical Poincaré line bundle by a gerbe/flux factor).

Example 5.1 (Explicit GL(1) calculation)**.** *Let $X = T^2$ and $D = \{p\}$ a single marked point with photonic phase $\mathrm{ph}_p = \exp(2\pi i\theta)$. Line bundles with unit-degree and photonic framing at p are parametrized by a torus $\mathrm{Pic}^1_{\mathrm{ph}}(T^2) \cong T^2$. Local systems with monodromy $(\alpha, \beta) \in (\mathbb{C}^\times)^2$ acquire a photonic twist multiplying monodromy around small loop at p by $\exp(2\pi i\theta)$. Fourier transform identifies delta-D-modules supported at points of $\mathrm{Pic}^1_{\mathrm{ph}}$ with characters (α, β) shifted by θ, matching eigenvalues.*

6 Approach: D-modules, perverse sheaves and categorical tools

6.1 Twisted D-modules and gerbe-twists

Photonic gerbe fluxes naturally produce twists of D-module categories (modules over twisted differential operators). The photonic Fourier–Mukai kernel is a twisted Poincaré sheaf (a line bundle up to a gerbe) implementing an integral transform between twisted D-modules and twisted quasi-coherent sheaves.

6.2 Spectral curve and Hitchin methods, an innovative strategy

For reductive G, one may consider a photonic Hitchin system where the Higgs fields are modified by photonic potentials; the spectral curve acquires extra parameters (photonic moduli) and the classical integrable system viewpoint yields spectral data (eigenvalues) encoding photonic insertions. Quantization of the Hitchin system (via opers or nonabelian Hodge) is expected to produce the spectral category on the right-hand side.

6.3 Compatibility with Hecke functors

Key constraints on any proposed equivalence (4.1) are compatibility with Hecke functors. On the automorphic side, Hecke functors modify bundles at points and transform D-modules; on the spectral side, they correspond to tensoring/quasi-coherent operations that change local monodromy. Verifying this compatibility in the photonic setting involves tracking how photonic Hecke modifications shift gerbe flux and local monodromy—tasks manageable in the GL(1) case and tractable for reductive groups using local-to-global factorization.

7 Crucial testing, implications and experimental signatures

7.1 Potential computational tests

- **GL(1) lattice checks:** discretize X and implement abelian Fourier transforms with photonic twist; verify matching of characters and Hecke shifts numerically.
- **Spectral curve computation:** compute photonic-deformed spectral curves for rank-2 examples and compare predicted eigenvalue shifts with numerical diagonalization of photonic operator families.

7.2 Physical observables

If the conjecture organizes photonic gauge sectors, observable predictions include:

- robustness of certain photonic spectral lines (Hecke eigenmodes) under adiabatic insertion of photonic defects;
- quantized responses (phase accumulations) tied to spectral parameters on the dual side;
- categorical constraints on mode conversions in metasurfaces controlled by photonic Hecke modifications.

8 Discussion and further directions

We propose to continue and research a conceptual scaffold linking Geometric Langlands ideas with photonic gauge phenomena. Future work directions:

1. rigorous construction of twisted photonic D-module categories and of $\mathrm{Loc}^{\mathrm{ph}}_{{}^LG}$ as derived stacks;
2. extension to wild ramification / Stokes phenomena modeling open photonic ends;
3. study of nonabelian examples (e.g. $G = \mathrm{SL}_2$) using quantization of photonic Hitchin systems.

Higher stacks and twisted D-modules remain of interest.

9 Stacky Sheaves, Photons and Cohomological Dualities

We collect the stack-theoretic foundations used in the main text and highlights how higher-stack language organizes photonic bundle data and their dualities.

9.1 Photonic stacks

We define a *photonic stack Ph* over the site of smooth manifolds whose objects over U are higher-geometric data:

$$(\mathcal{E}, \nabla, \mathcal{G}, \kappa),$$

where $\mathcal{E}$ is a principal G-bundle with connection ∇, $\mathcal{G}$ is a p-gerbe with connection (modeling higher-form photonic flux), and κ denotes a collection of local coupling forms controlling interaction between degrees. Morphisms are gauge transformations together with higher homotopies (1- and 2-morphisms) respecting the gerbe structure.

Remark 9.1. *The stack Ph is naturally a derived stack when one includes deformation theory of connections and gerbes; obstruction classes live in higher hypercohomology groups capturing the photonic coupling constraints.*

9.2 Twists and modules

Given *Ph*, one may form categories of (twisted) sheaves or modules on it: quasi-coherent complexes, D-modules twisted by gerbes, and categories of local systems with photonic twists. These categories carry natural monoidal structures coming from tensoring of line bundle components and convolution-type constructions induced by Hecke correspondences.

9.3 Cohomological dualities

Duality statements arise by pairing photonic stacks with dual stacks parameterizing complementary data (electric vs magnetic sectors). Formally, there is a Poincaré-type duality pairing between hypercohomology classes controlling gerbe fluxes and cycles supporting dual local systems. Under suitable finiteness hypotheses this pairing induces Fourier–Mukai transforms between derived categories associated to dual photonic stacks, providing a categorical manifestation of photonic cohomological duality.

10 Unobserved Photonic States

Let us now explore a class of photonic configurations that are *locally invisible* to ordinary line-bundle holonomy measurements but are detected by higher-gerbe invariants; we shall call these *unobserved photonic states.*

10.1 Definition and examples

An unobserved photonic state on a manifold M is a photonic configuration whose projection to ordinary first Chern-class data vanishes but whose higher differential cohomology (gerbe) class is nontrivial. Concretely, a $U(1)$-gerbe with curvature H that integrates nontrivially over 3-cycles but whose associated induced monopole charges on 2-cycles vanish provides a primary example.

10.2 Detection and probes

Detection of such states may be challenging and requires probes sensitive to higher-holonomy:

- surface holonomy measurements (integrating gerbe connections over embedded surfaces);
- interferometric protocols coupling loops to surfaces (higher Aharonov–Bohm-type experiments);
- spectral flow diagnostics in photonic band structures when adiabatically varying a higher-form flux parameter.

10.3 Role in categorical correspondences

Unobserved photonic states furnish nontrivial twistings of categories on both automorphic and spectral sides: they modify moduli of objects (shifting gerbe-twists) without changing underlying bundle topological type. In the Langlands-style correspondence proposed above, these twists must be tracked as part of the photonic data to realize full equivalences; failure to include them leads to mismatches in spectral multiplicities and Hecke eigenvalues.

11 Photonic Geometric Langlands Notes

We provide technical heuristics specifically aligned with adapting Geometric Langlands machinery to the photonic setting. This should aid research.

11.1 Twisted Fourier–Mukai kernels

The core analytic ingredient in the abelian case is a Poincaré kernel modified by a gerbe class. Let $\mathcal{P}$ be the classical Poincaré bundle on $\mathrm{Pic} \times \mathrm{Loc}$. In the presence of gerbe flux $\mathcal{G}$ the kernel is replaced by a *gerbe-twisted Poincaré object* $\widetilde{\mathcal{P}}$ living in an appropriate derived category of twisted sheaves; explicit local formulas use Čech representatives of the gerbe and local connection data.

11.2 Twisted opers and photonic opers

On the spectral side, opers (differential operators encoding flat connections with extra structure) admit gerbe-twisted variants: the local differential equations acquire extra potential terms coming from photonic coupling forms. Quantization of these twisted opers is expected to produce eigenvalue problems matching photonic Hecke eigen-D-modules under the integral transform.

11.3 Perverse t-structures and photonic perversities

Choice of a t-structure on the automorphic side is delicate once photonic defects are present. We propose *photonic perversities* that assign shifts to strata determined by defect depth and gerbe-degree; these choices are constrained by compatibility with Verdier duality and the expected support conditions of Hecke eigenobjects.

11.4 Practical heuristics for checking the conjecture

For (further) concrete work, in theory the following could be considered:

1. Verify the GL(1) transform explicitly on small lattices and with discrete gerbe representatives.
2. Construct local models of photonic Hecke modifications and check their action on twisted D-modules in examples (rank 1 and rank 2).
3. Trace how gerbe-twists modify the classical Hitchin fibration and compute low-rank examples of photonic spectral curves numerically.